Module 2: Review of Fluid Mechanics Basic Principles for Water Resources Engineering

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Basic Definitions

- **Mass** – quantity of matter that a substance contains
- **Weight** – effect of the force of gravity upon a substance
- **Density** – mass of a substance per unit volume
- **Viscosity** – measure of a fluid's resistance to flow
- **Mass Density (density) (ρ):**

  For water at 4°C, $\rho = 1000\ \mathrm{kg/m^3}$ or $1.94\ \mathrm{slugs/ft^3}$.
- **Specific weight (γ):** gravitational force per unit volume of water.

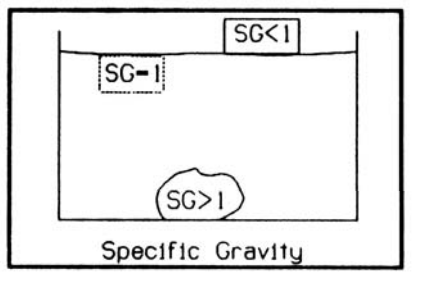
  For water at 4°C, $\gamma = 9810\ \mathrm{N/m^3}$ or $62.4\ \mathrm{lb/ft^3}$.

  $$\gamma = \rho g$$

  where g = gravitational constant

Basic Definitions

- **Specific Gravity:** $$\text{Specific Gravity} = \frac{\text{Density of substance}}{\text{Density of water}}$$



Hydraulics for Operators, Barbara Hauser, Lewis Publishers, Boca Raton, FL. 1993.

Basic Definitions

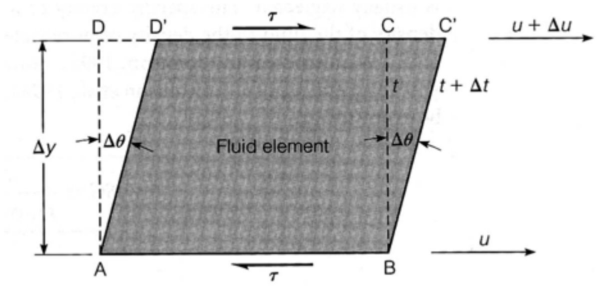
$$\gamma = \rho g$$

- Specific weight of a fluid is defined as the weight per unit volume and is related to density.
- Specific gravity of a fluid is the ratio of the density of the fluid to the density of pure water at standard conditions (such as at 15°C).

$$specific\ gravity = \frac{\rho_{fluid}}{\rho_{H_2 0 @ 15^o C}}$$

Newton's equation of viscosity: $\tau = \mu \frac{d\mu}{dy}$

Viscosity is the proportionality constant between shear stress and strain rate of a fluid element:



Kinematic viscosity related to (dynamic) viscosity: $\nu = \frac{\mu}{\rho}$

Basic Definitions

- **Dynamic Viscosity (μ):** ratio of the shear stress to the velocity gradient between two thin sheets of fluid
- **Kinematic Viscosity (ν):** ratio of the dynamic viscosity to the density

- Fluid characterized by:
  - Temperature, T
  - Density, ρ (units: mass/volume)
  - Absolute/Dynamic Viscosity, μ (units: force-time/area)

Basic Definitions

Fluid flow characterized by:

- Velocity, V (units: length/time)
- Kinematic Viscosity, ν (units: area/time)

$$\boxed{\nu = \frac{\mu}{\rho}}$$

Where ν = kinematic viscosity
μ = absolute/dynamic viscosity
ρ = fluid density

Basic Definitions

Example:

According to the table of the physical properties of water, the specific weight of water at 20°C is 9789 N/m^3, what is its density?

$$\rho = \frac{\gamma}{g} = \frac{9789\ \text{N/m}^3}{9.81\ \text{m/sec}^2}$$

$$= 998\ \text{N} - \text{sec}^2/\text{m}^4 \left(\frac{1\,\text{kg} - \text{m/sec}^2}{\text{N}} \right)$$

$$= 998\ \text{kg/m}^3$$

Properties of Water in U.S. Customary (English) Units

Temp. (°F)	Specific Weight γ (lb/ft^3)	Mass Density ρ (lb-sec^2/ft^{4}or slugs/ft^3)	Absolute Viscosity μ (x 10^5 lb-sec/ft^2)	Kinematic Viscosity ν (x10^5 sec/ft^2)	Vapor Pressure p_v (psi)
32	62.42	1.940	3.746	1.931	0.09
40	62.43	1.940	3.229	1.664	0.12
50	62.41	1.940	2.735	1.410	0.18
60	62.37	1.938	2.359	1.217	0.26
70	62.30	1.936	2.050	1.059	0.36
80	62.22	1.934	1.799	0.930	0.51
100	62.00	1.927	1.424	0.739	0.95
120	61.71	1.918	1.168	0.609	1.69
140	61.38	1.908	0.981	0.514	2.89
160	61.00	1.896	0.838	0.442	4.74
180	60.80	1.890	0.780	0.413	5.99
212	59.83	1.860	0.593	0.319	14.70

From: George Tchobanoglous (Metcalf & Eddy, Inc.). *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc., New York, NY. 1981. Table B-2.
Warren Viessman, Jr. and Mark J. Hammer. *Water Supply and Pollution Control, Sixth Edition.* Addison Wesley, Menlo Park, CA. 1998. Table A.8.
Larry W. Mays. *Water Resources Engineering, 1st Edition*. John Wiley & Sons, Inc. New York, NY. Table 2.1.1.

Properties of Water in SI (Metric) Units

Temp. (°C)	Specific Weight γ (kN/m^3)	Mass Density ρ (kg/m^3)	Absolute Viscosity μ (x 10^3 kg/m-sec or 10^3 N-sec/m^2)	Kinematic Viscosity ν (x10^6 m^2/sec)	Vapor Pressure p_v (kPa)
0	9.805	999.8	1.781	1.785	0.61
5	9.807	1000.0	1.518	1.519	0.87
10	9.804	999.7	1.307	1.306	1.23
15	9.798	999.1	1.139	1.139	1.70
20	9.789	998.2	1.002	1.003	2.34
25	9.777	997.0	0.890	0.893	3.17
30	9.764	995.7	0.798	0.800	4.24
40	9.730	992.2	0.653	0.658	7.38
60	9.642	983.2	0.466	0.474	19.92
80	9.530	971.8	0.354	0.364	47.34
100	9.399	958.4	0.282	0.294	101.33

From: George Tchobanoglous (Metcalf & Eddy, Inc.). *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc., New York, NY. 1981. Table B-2.
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Basic Definitions

Reynolds Number, Re or N_R:

$$\mathrm{Re} = N_R = \frac{Vd\rho}{\mu} = \frac{Vd}{\nu}$$

Where V = fluid velocity
D or d = diameter of equivalent pipe
ρ = fluid density
μ = absolute viscosity
ν = kinematic viscosity

- THE REYNOLDS NUMBER SHOULD BE DIMENSIONLESS:
  - All units must cancel out!!

Basic Definitions

Example:

- Calculate the Reynolds number for the following water flow conditions:
  V = 0.10 ft/sec
  D = 1 inch

Solution:

- Convert the pipe diameter to feet since V is in ft/sec.
  D = 1 inch/(12 in/ft) = 0.083 ft
- Since T is not given, assume T = 70°F, and look up the density, absolute viscosity, and kinematic viscosity at 70°F.
  ρ = 1.936 lb-sec^2/ft^4
  μ = 2.050 x 10^{-5} lb-sec/ft^2
  ν = 1.059 x 10^{-5} ft^2/sec

Basic Definitions

Solution:

Substituting:

$$Re = \frac{(0.10\, ft/\sec)(0.083\, ft)(1.936 lb - \sec^2/ ft^4)}{2.050x10^{-5} lb - \sec/ ft^2}$$

$$Re = 784$$

OR

$$Re = \frac{(0.10\, ft/\sec)(0.083\, ft)}{1.059x10^{-5}\ ft^2/\sec}$$

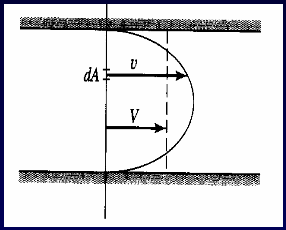
$$Re = 784$$

Flow Regimes

- ***Steady Flow*** – assumes constant flow rate throughout the analysis.
  - Flow velocity does not change with respect to time at a given location.
- ***Unsteady*** *Flow* – does not assume a constant flow rate throughout the analysis.
  - Flow velocity likely does change with respect to time at a given location.
- Can assume Steady Flow for the analyses performed in this class. Unsteady flow assumptions used for floodplain analysis, for example.

Flow Regimes

- Velocity Distribution in Pipe and Open-Channel Flow



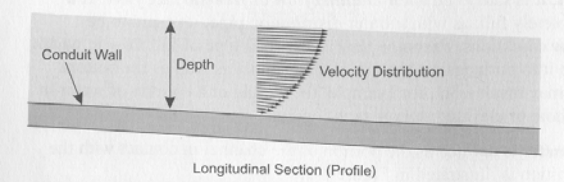
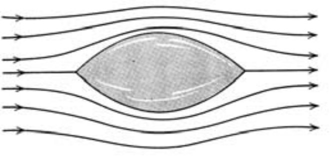
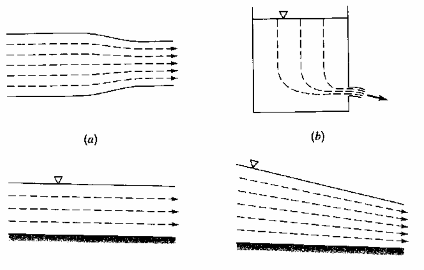


Figure 1-2: Velocity Distribution

Flow Regimes

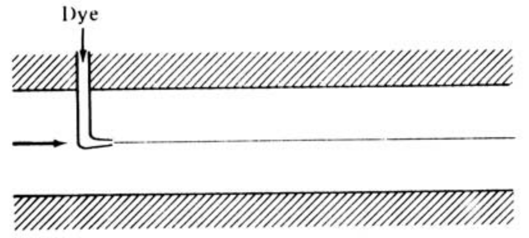
- ***Streamlines*** **–** paths of single fluid particles.





Flow Regimes

- **Laminar Flow**
  - Streamlines are parallel, smooth and predictable
  - No mixing occurs between layers
  - Reynolds number < 2100 to 4000



Flow Regimes

Example:

What is the maximum velocity (ft/sec) for laminar flow (assuming Re = 4000) in a 1-inch pipe (water is the fluid, 60ºF is the temperature)?

$$\mathrm{Re} = \frac{Vd\rho}{\mu}$$

Substituting :

$$4000 = \frac{V(1\,\mathrm{inch})(1\,\mathrm{ft}/12\,\mathrm{in})(1.934\,\mathrm{lb\text{-}sec^2/ft^4})}{2.359 \times 10^{-5}\,\mathrm{lb\text{-}sec/ft^2}}$$

$$V = 0.59\ \mathrm{ft/sec}$$

Maximum Velocity of Laminar Flow or Water in Circular Pipes

(Re = N_R= 4000)

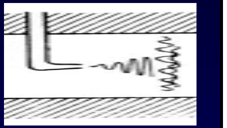
Diameter		Maximum Velocity of Laminar Flow	
millimeters	inches	meters/sec	feet/sec
6	0.25	0.71	2.34
12	0.5	0.36	1.17
25	1	0.18	0.59
50	2	0.09	0.29
150	6	0.03	0.10
300	12	0.01	0.05

Reference: McGhee, Chapter 3

Flow Regimes

Turbulent Flow **(calculated as steady flow although not)**

- Streamlines are not parallel or predictable
- Mixing occurs across pipe diameter
- Reynolds number > 2100 to 4000
- Eddies may occur in flow profile



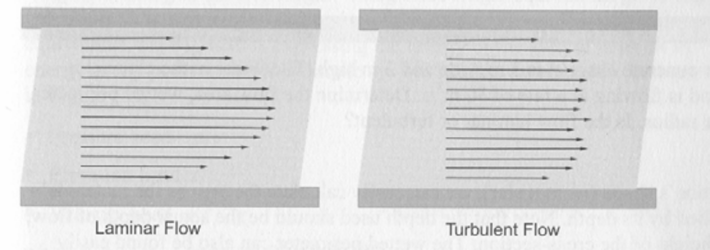
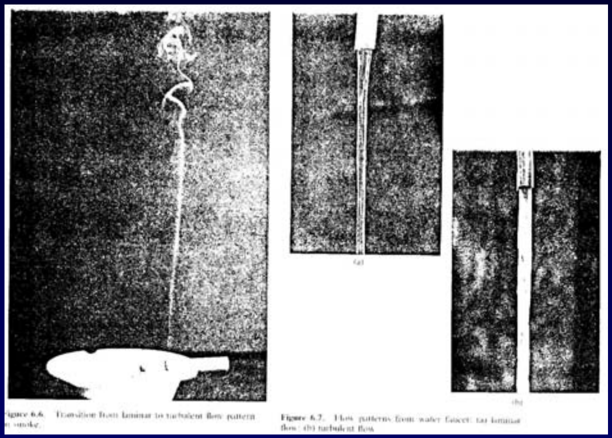


Figure 1-3: Instantaneous Velocity Distributions for Laminar and Turbulent Flow

Flow Regimes



From: *Introduction to Fluid Mechanics, Third Edition*. James E. John and William L. Haberman. Prentice-Hall, Englewood Cliffs, NJ. 1988.

Flow Regimes

Example:

Water flows full in a 5-foot diameter pipe at a velocity of 10 ft/sec. What is the Reynolds number? The temperature is 50°F.

- From table, at 50°F, $\nu = 1.41 \times 10^{-5}$ ft^2/sec.

$$\mathrm{Re} = \frac{VD}{\nu}$$

$$\mathrm{Re} = \frac{(10\, ft/\sec)(5\, ft)}{1.41x10^{-5}\ ft^2/\sec}$$

$$\mathrm{Re} = 3.55x10^6$$